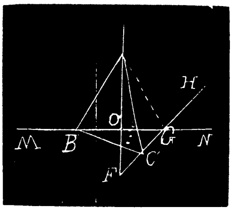
or, $\dfrac{h(\sin C\sin E-\cos C\sin E)}{y(\sin A\cos E+\cos A\sin E)}=\dfrac{\sin C}{\sin A}$; or $h(1-\cot C\tan E)=y(1+\cot A\tan E)$, or

$$h-y=(h\cot C+y\cot A)\tan E\text{; but }\tan E=(h-y)/x,$$

$\therefore$ $x=y\cot A+h\cot C$, or $y=x\tan A-h\cot C\tan A$, as the locus of C.



For $C=A$, we have $y=x\tan A-h$.

Make $OF=OA$, draw FH so as to make $\angle HGN=\angle A$, draw AG. $\angle HGN=OGF=\angle AGO=\angle A$.

Since $\angle BAC=\angle BGC$, $ABCG$ is concyclic.

$\therefore$ $\angle ACB=\angle AGB=\angle A$, $\therefore$ $\angle C=\angle A$, which unifies geometrically for the case in which the angles at A and C remain equal.

92. Proprosed by **JOSIAH H. DRUMMOND, LL. D., Counselor at Law, Portland, Me.**

Let $ABCD$ be a quadrilateral inscribed in a circle. Draw the diagonals AC and BD. Show that $AB.BC : DC.AD=BD : AC$. [From a note in *Young's Geometry*, edition of 1830.]

Solution by the PROPOSER, and J. SCHEFFER, Hagerstown, Md.

Let F be the intersection of the diagonals.

Then $AB : BF :: CD : CF$,

or $AB : CD :: BF : CF$,

and $BC : AD :: CF : DF$.

Hence $AB.BC : AD.CD :: BF : DF$,

(I) and $AB.BC+AD.CD : AD.CD : BF+DF(=BD) : DF$.

In like manner it is shown

(II) that $AB.AD+BC.CD : BC.DC :: AC : CF$.

But $AD : DF :: BC : CF$,

or $AD.CD : DF :: BC.CD : CF$.

Combining these with (I) and (II), we have

$$AB.BC+AD.CD : AB.AD+BC.CD :: BD : AC. \qquad \text{Q. E. D.}$$

Also solved by *B. F. SINE*, *CHAS. C. CROSS*, *WALTER H. DRANE*, and *G. B. M. ZERR*.

93. Proposed by **G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.**

While surveying in a level field I notice a mountain behind a hill. Wishing to know the height of each I take the angles of elevation of the tops of both and find them to $\beta=45°$, $\delta=40°$. I then measure a straight line $a=400$ feet, and find the angles of elevation of the tops to be $\gamma=42°$, $\mu=38°$. After measuring $b=300$ feet more in the same straight line I find the elevations to be $\lambda=40°$, $\nu=36°$. Find the height of each.

Solution by the PROPOSER.

Let $AB=400$ feet$=a$, $BC=300$ feet$=b$, $OP=x$, $QR=y$.

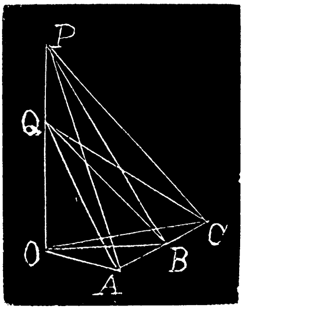
$\angle OAP=45°=\beta$, $\angle OAQ=40°=\delta$, $\angle OBP=42°=\gamma$, $\angle OBQ=38°=\mu$, $\angle OCP=40°=\lambda$, $\angle OCQ=36°=\nu$.

$OA=m$, $OB=n$, $OC=p$, $\angle OCA=\theta$.

$\therefore$ $m=y\cot\delta=x\cot\beta$, $n=y\cot\mu=x\cot\gamma$.

$p=y\cot\nu=x\cot\lambda$.

Also from triangles OCA and OCB,



$m^2=p^2+(a+b)^2-2p(a+b)\cos\theta$ ....(1).

$n^2=p^2+b^2-2pb\cos\theta$ ....(2).

The values of m, n, p in (1) and (2) and eliminating $\cos\theta$, we get

$$x=\sqrt{\frac{ab(a+b)}{b\cot^2\beta+a\cot^2\lambda-(a+b)\cot^2\gamma}} \qquad y=\sqrt{\frac{ab(a+b)}{b\cot^2\delta+a\cot^2\nu-(a+b)\cot^2\mu}}$$

Substituting we get $y=1505.183$ feet, $x=4232.505$ feet.

Also solved by *ALOIS F. KAVORIK*, and *CHAS. C. CROSS*.

CALCULUS.

71. Proposed by J. C. CORBIN, Pine Bluff, Ark.

Form the differential equation of the third order, of which $y=c_1e^{2x}+c_2e^{-3x}+c_3e^x$ is the complete primitive.

III. Solution by C. HORNUNG, A. M., Professor of Mathematics, Heidelberg University, Tiffin, O.; COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.; C. W. M. BLACK, A.M., Professor of Mathematics, Wesleyan Academy, Wilbraham, Mass.; G. B. M. ZERR, A. M., Ph. D., The Russell College, Lebanon, Va.; and the PROPOSER.

$y=c_1e^{2x}+c_2e^{-3x}+c_3e^x$ .....(1), $dy/dx=2c_1e^{2x}-3c_2e^{-3x}+c_3e^x$........(2);

$d^2y/dx^2=4c_1e^{2x}+9c_2e^{-3x}+c_3e^x$....(3). $d^3y/dx^3=8c_1e^{2x}-27c_2e^{-3x}+c_3e^x$....(4).

(2)−(1) gives $dy/dx-y=c_1e^{2x}-4c_2e^{-3x}$ ....(5),

(4)−(1) gives $d^3y/dx^3-y=7c_1e^{2x}-28c_2e^{-3x}$ ....(6),

(6)−7(5) gives $d^3y/dx^3-7(dy/dx)+6y=0$.

IV. Solution by J. SCHEFFER, A. M., Hagerstown, Md.

The equation reduces itself to what is the equation the three roots of which are 1, 2, −3. This equation is $z^2-7z+6=0$; consequently the complete primitive is $d^3y/dx^3-7(dy/dx)+6y=0$.

72. Proposed by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.

A man has a park in the form of a parabolic segment cut off by a chord making an angle $\frac{1}{4}\pi$ with the axis. Within the park is a right angled triangular flower plat with one vertex at the center of gravity of the segment, the other vertex at the lower extremity of the chord, and the right angle on the diameter bisecting the chord. The park contains 30